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B.A./B.Sc. (Part-I) Examination, 2018

MATHEMATICS

First Paper

BMG-101

(Algebra & Trigonometry)

Time : Three Hours] *[Maximum Marks : 65*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. Give answer of each part in about 50 words.

$1\frac{1}{2} \times 10 = 15$

1. (i) Define the Cauchy sequence.
   (ii) Show that the set N of natural numbers is not group with respect to binary operation '+'.
   (iii) Find the general value of log i.
   (iv) Give an example of a ring with zero-divisor.

(v) Define convergent and divergent infinite series with an example.

(vi) Define permutation group.

(vii) Define group homomorphism.

(viii) Prove that identity element of a group is unique.

(ix) Define a division ring.

(x) State De Moivere's theorem.

Section-B

(Short Answer Type Questions)

Note : Attempt **all** questions. Give answer of each question in about 200 words. $6 \times 5 = 30$

2. Show that the four fourth roots of unity namely 1, –1, i, –i form a group w.r.t. multiplication.

OR

Prove that

$$(ab)^{-1} = b^{-1}a^{-1}, \ \forall\, a,b \in G$$

i.e. the inverse of product of two elements in a group G is the product of inverses in the reverse order.

3. Prove that intersection of two subrings is a subring.

OR

Prove that every subgroup of a cyclic group is cyclic.

4. Examine the convergence of the series:

$$\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+----$$

OR

Show that the set of integers does not form a group with respect to binary operation multiplication.

5. If $\text{Sin}(\theta+i\phi)=\tan\alpha+i\sec\alpha$ then prove that

$\cos 2\theta \cosh 2\phi=3$

OR

If (R, +, .) be a ring. Show that

(i) $a.0=0.a=0$

(ii) $(-a).b=-(ab)$

6. Prove that order of an element of a group is the same as that of its inverse a^{-1}.

OR

$$i\log\frac{x-i}{x+i}=\pi-2\tan^{-1}x$$

Section-C

(Long Answer Type Questions)

Note : Attempt any **two** questions. Give answer of each question in about 500 words.

$10 \times 2 = 20$

7. Prove that every finite integral-domains is a field.

8. Express $\tan^{-1}(x+iy)$ as the sum of real and imaginary parts.

9. A non-empty subset S of a ring R is subring if and only if

   (i) $a, b \in S \Rightarrow a - b \in S$

   (ii) $a, b \in S \Rightarrow ab \in S$

10. Sum the series

$$\tan^{-1}\frac{1}{1+1.2} + \tan^{-1}\frac{1}{1+2.3} + \tan^{-1}\frac{1}{1+3.4} + \text{---to n terms.}$$

11. Prove that

$$\frac{\pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + ---$$