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B.A./B.Sc. (Part-III)

Examination, 2022

MATHEMATICS

Second : Paper

BMG-302

(Complex Analysis)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. 1½×10=15

1. (a) Define analytic function with example.

(b) State Cauchy- Riemann equation.

(c) Define bilinear Transformation.

P.T.O.

(d) Define conformal mapping .

(e) Write down the Cauchy's fundamental Theorem.

(f) State Morera's theorem.

(g) Define analytic continuation.

(h) Define removable singularity with example.

(i) Find the residue of $f(z) = \frac{z^4}{(z^2 + a^2)}$ at $z = ai$

(j) Define zeros of an analytic function.

Section-B

(Short Answer Type Questions)

Note : Attempt **all** questions. 8×5=40

2. State and prove Cauchy-Riemann equations in polar form.

OR

Prove that e^{-x} (x cos y+ y sin y) is harmonic and find harmonic conjugate.

6. Find the residue of

$$\frac{z^3}{(z-1)^4(z-2)(z-3)} \text{ at } z=1$$

OR

State and prove Rouche's theorem.

Section-C

(Long Answer Type Questions)

Note : Attempt any **two** questions. $10\times2=20$

7. If $u+v = \frac{2\sin 2x}{e^{2x}+e^{-2y}-2\cos 2x}$ and $f(z) = u+iv$ is an analytic function of z, then find $f(z)$ in terms of z.

8. Find all bilinear transformation which transform the unit circle $|z| \leq |$ into the unit circle $|w| \leq |$.

9. State and prove Taylor's theorem.

10. State and prove Cauchy's residue theorem.

11. State and prove sufficient condition for f(z) to be analytic.

3. If the Mapping $w = f(z)$ is conformal, then show that f(z) is an analytic function of z.

OR

State and prove Cauchy's integral formula.

4. Find the bilinear transformation which transform the points $z = 2, i, -2$ into the points $w = 1, i, -1$ respectively.

OR

Prove that the Cross-ratio remains invariant under a bilinear transformation.

5. Find the kind of singularity of the following functions :

(a) $f(z) = \tan\frac{1}{z}$ at $z = 0$

(b) $f(z) = \frac{z}{1+z^4}$

OR

State and prove fundamental theorem of algebra.