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B.A./B.Sc. (Part-III) Examination, 2022

Mathematics

First Paper

BMG-301

(Real Analysis)

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. Give answer of each part in about 50 words.

$1\frac{1}{2} \times 10 = 15$

P.T.O.

1. (a) Prove that if M and N are neighbourhoods of a point x, then M-N is also neighbourhood of x.

(b) Find the limit points of the set

$S = \{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3} \ldots\ldots\}$

(c) Find the limit superior and limit inferior of the sequence.

$\langle 1,3,5,\ 1,3,5,\ 1,3,5,\ \ldots\ldots \rangle$

(d) Define uniform convergence of sequences of functions.

(e) Write the statement of Darboux theorem.

(f) Find $\lim_{(x,y)\to(0,0)} y \sin \frac{1}{x}$

(g) If $f(x)=x$, $\forall x \in [0,3]$ and $P=\{0,1,2,3\}$ be a partition of P then find $L(p,f)$ and $\mu(P,f)$.

(h) Test the convergence of the integral

$$\int_0^1 \frac{dx}{x^3(1+x^2)}$$

(i) Define open set in a metric space.

(j) Define complete metric space.

Section-B

(Short Answer Type Questions)

Note : Attempt **all** questions. Give answer of each question in about 200 words.

8×5=40

2. Prove that the unit interval [0,1] is uncountable.

OR

Prove that the intersection of an arbitrary collection of closed sets is closed and union of finite collection of closed sets is closed.

3. Prove that every Cauchy sequence is bounded but the converse need not be true.

OR

Show that the sequence $<f_n>$ where $f_n(x) = \frac{nx}{1+n^2x^2}$ is not uniformly convergent on R.

4. Prove that if a function f is continuous on the closed interval [a,b], then it attains its supremum and infimum at least once in [a,b].

OR

Prove that the function $f(x)=x^2\ \forall x\in R$ is uniformly continuous in any interval but is not uniformly continuous on R. Where R is the set of real numbers.

5. Prove that if f is continuous on [a,b] then f is R-integrable on [a,b].

OR

Test the convergence of $\int_0^\infty e^{-x}\frac{\text{sim}\,x}{x}dx$.

6. Prove that in any metric space X every open sphere is an open set.

OR

Show that the set C of complex numbers with usual metric is a complete metric space.

Section-C

(Long Answer Type Questions)

Note : Attempt any **two** questions. Give answer of each question in about 500 words.

10×2=20

7. Let A and B be any two subsets of R, then prove that

$D(A\cup B)=D(A)\cup D(B)$

8. Prove that a series $\Sigma un(x)$ of real valued functions defined on an interval I is uniformly convergent on I if there exist a convergent series $\sum M_n$ of positive constants such that $|\mu n(x)|\le M_n\ \forall\ n=1,2,3\ldots\ldots$ and $\forall x\in I$

9. Find the maxima and minima of $u=x^2+y^2+z^2$ subject to the conditions $ax^2+by^2+cz^2=1$ and $lx+my+xz=0$.

10. Discuss the convergence of the gamma function. $\int_0^\infty x^{n-1} e^{-x} dx$

11. Let x be a metric space If $<x_n>$ and $<y_n>$ are sequences in X such that $x_n \to x$ and $y_n \to y$, then prove that the sequence $<d(x_n, y_n)>$ of real numbers converges to $d(x,y)$.