15/122-BC

B. C. A. (Second Semester) (Regular/Back) Examination, 2015

Paper - Fourth

BCA-204-Discrete Mathematics

Time : Three Hours] *[Maximum Marks : 75*

Note : Attempt questions from ***all*** Sections as per instructions.

SECTION – A
(Very Short Answer Type Questions)

Note : Attempt ***all*** parts of this question. Give answer of each part in about **50** words. $1\frac{1}{2} \times 10 = 15$

1. (i) Define Cartesian product of two sets with an example.

(ii) Define group with the help of an example.

(iii) State the relation and types of relation.

(iv) Define logical conjunction and disjunction.

(v) Define a simple graph with an example.

(vi) Define sub graph and isomorphic graph.

(vii) State Euler path and Euler circuit.

(viii) Show that every self complementary graph has 4K or 4K + 1 vertices.

(ix) Define tree and their properties.

(x) Write the Fulkerson algorithm of network flow.

SECTION – B
(Short Answer Type Questions)

Note : Attempt all questions. Give answer of each question in about **200** words. $8 \times 5 = 40$

2. If R is a relation from A to B, S is a relation from B to C and T is a relation from C to D then show that :

$$(ROS)\,OT = RO(SOT).$$

OR

The necessary and sufficient condition for a non-empty subset H of a group (G, *) to be a subgroup is $a \in H, b \in H \Rightarrow a * b^{-1} \in H$ where b^{-1} is the inverse of b in G.

3. Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

OR

Show that ~ r is a valid conclusion from the premises. $p \Rightarrow \sim q, r \Rightarrow p, q$ with truth table.

4. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

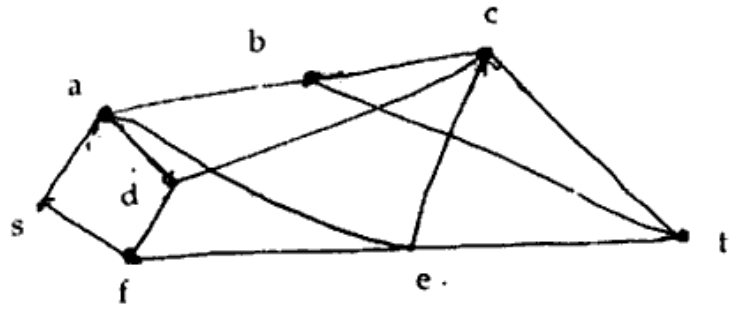
OR

Prove that the sum of degrees of the vertices in an undirected graph is even.

5. Give an example of a graph which is Hamiltonian but not non-Eulerian.

OR

Find the shortest path from vertex s to t and its length from the graph given below :



6. A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree 1 does it have ?

OR

A simple graph G has a spanning tree if and only if G is connected.

SECTION – C

(Long Answer Type Questions)

Note : Attempt any ***two*** questions. Give answer of each question in about **500** words. $10 \times 2 = 20$

7. Set $(G,*)$ and $(G_1,*_1)$ be two groups and let $f : G \to G_1$ be a homomorphism from G to G_1 then :

(a) $f(e) = e_1$ where e is the identity in G and e_1 is the identity in G_1.

(b) $f\left(a^{-1}\right) = (f(a))^{-1}$ for all $a \in G$.

(c) If H is a subgroup of G then $f(H) = \{f(h) : h \in H\}$ is a subgroup of G_1.

8. For the set $I_4 = \{0, 1, 3\}$ show that the modulo 4 system is a ring.

9. Write short notes on the following :

(a) Path and circuits

(b) Shortest path problem.

10. Write short notes on the following :

(a) Hamiltonian graph.

(b) Travelling salesman problem.

11. (a) A full m-ary tree with i internal vertex has n = mi + 1 vertices.

(b) There are atmost m^h leaves in an m-ary tree of height h.