B.Sc. (Part-I) EXAMINATION, 2015
MATHEMATICS
Paper First : Algebra and Trigonometry

Note : Answer questions from all Sections as per instructions.

Section–A (Very Short Answer Type Questions)

Attempt all parts of this question. Give answer of each part in about 50 words. $1^{1/2} \times 10 = 15$

1. (i) Define a group with example.
   (ii) Resolve $\sin^2 (x + iy)$ into real and imaginary parts.
   (iii) What is the difference between Normal Subgroup of group and cosets of a group.
   (iv) Define convergent and divergent of infinite series with an example.
   (v) Define order of an element in additive group with an example.
   (vi) Define odd and even per mutation with example.
   (vii) What is difference between sequence and bounded sequence explain with example.
   (viii) Give an example of a ring with zero divisors.
   (ix) In a ring $(\{0, 1, 2, 3, 4\}, +_5, x_5)$ find the additive Inverse of 2 and 3.
   (x) Define the characteristic of a ring with an example.

Section–B (Short Answer Type Questions)

Attempt all questions. Give answer of each question in about 200 words. $6 \times 5 = 30$

2. Test the convergence of the series whose nth term is :

$$\frac{n^4 + 1 - n^2}{}$$

Or

Show that if a sequence is convergent then its limit is unique.

3. If $(G, 0)$ is a group then show that :
   (i) The identity element of G is unique.
   (ii) $(a \, o \, b)^{-1} = b^{-1} \, o \, a^{-1} \; \forall \; a, b \in G$

Or

If $\tan(\theta + i\varphi) = \tan 2 + i \sec 2$. Prove that :

(i) $e^{2\varphi} = \pm \cot\frac{1}{2}2$ (ii) $2\theta = n\pi + \frac{1}{2}\pi + 2$

4. The union of two subgroup of a group G is a subgroup of G if one is contained in the other.

Find the sum of the following series : Or

$$\sec\theta \sec 2\theta + \sec 2\theta \sec 3\theta + \sec 3\theta \sec 4\theta + n \text{ terms}$$

5. State and prove the Lagrange's Theorem in group. Or

If φ lies between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ show that

$$\varphi = \frac{\pi}{2}\cot\varphi + \frac{1}{3}\cot^3\varphi - \frac{1}{5}\cot^5\varphi + \ldots.$$

6. Prove that :

$$i \log\frac{x-i}{x+i} = \pi - 2\tan^{-1}x$$

Or

The set of residue classes (modulo 6) forms a commutative ring with unity with respect to addition and multiplication of residue classes (mod 6).

Section–C (Long Answer Type Questions)

Attempt any two questions. Give answer of each question in about 500 words. $10 \times 2 = 20$

7. Prove that :

$$\tan\left(i \log\frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2-b^2}$$

8. A non empty subset S of a ring R is a subring if and only if :
   (i) $a, b \in S \Rightarrow a - b \in S$
   (ii) $a, b \in S \Rightarrow ab \in S$

9. Separate $\sin^{-1}(\cos\theta + i\sin\theta)$ into real and imaginary parts where $0 < \theta < \pi$.

10. Test the convergence of the series

$$\frac{2}{\beta} + \frac{1+2}{1+\beta} + \frac{(1+2)(2+2)}{(1+\beta)(2+\beta)} + \ldots.$$

11. Show that the relation "Congruence modulo m" on set z of all integers is an equivalence relation.