

B.Sc. (Part-I) EXAMINATION, 2018
MATHEMATICS
Paper Second : Calculus

Note : Answer questions from all Sections as per instructions.

Section-A (Very Short Answer Type Questions)

Attempt all parts of this question. Give answer of each part in about 50 words. $1\frac{1}{2} \times 10 = 15$

- (i) Write down n^{th} derivative of $(ax + b) - 1$.
- (ii) Show that $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$ does not exist.
- (iii) Explain Geometrical meaning of Lagrange Mean Value Theorem.
- (iv) Find the n^{th} derivative of $x^3 \cos x$.
- (v) Expand $\log \sin(x + h)$ in power of h by Taylor's theorem.
- (vi) If $u = f(y - z, z - x, x - y)$ prove that :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

(vii) Use reduction formula to integrate $\sin^{1/2} x \cos^{7/2} x$.

(viii) Evaluate the value of following integral $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$.

(ix) Find whole area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(x) Evaluate :

$$\int_1^2 \int_0^x \frac{dx dy}{x^2 + y^2}$$

Section-B (Short Answer Type Questions)

Attempt all questions. Give answer of each question in about 200 words. $6 \times 5 = 30$

2. Examine the continuity of the function :

$$f(x) = \begin{cases} -x^2 & \text{if } x \leq 0 \\ 5x - y & \text{if } 0 \leq x \leq 1 \\ 4x^2 - 3x & \text{if } 1 < x < 2 \\ 3x + y & \text{if } x \geq 2 \end{cases}$$

State and prove Leibnitz's Theorem.

Or

3. If $y = a \cos(\log x) + b \sin(\log x)$ then prove the following :

$$x^2 y_{n+2} + (2n + 1) x y_{n+1} + (n^2 + 1) y_n = 0 \quad \text{Or}$$

$$\text{Find } \lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$$

4. If $x^x y^y z^z = c$ then show that at $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = -[x \log ex]^{-1} \quad \text{Or}$$

If $u = f(y/x)$ show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0.$$

5. Find maximum or minimum values of function $x^3 y^2 (1 - x - y)$. Or
Find the area included between the curve $x = a(t + \sin t)$,
 $y = a(1 - \cos t)$.

6. Show that integral $\int \int \int x^{l-1} y^{m-1} z^{n-1} dx dy dz$ integrand over region
in first octant below the surface

$$\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r = 1 \quad \text{Or}$$

Transform the following integral

$$\int_0^2 \int_0^{\sqrt{2x-1}} \frac{x \, dx \, dy}{\sqrt{dx^2 + dy^2}}$$

by changing to Polar Co-ordinate hence evaluate it.

Section - C (Long Answer Type Questions)

Attempt any two questions. Give answer of each question in about 800 words. $10 \times 2 = 20$

Evaluate $\int \int \int (x + y + z) dx dy dz$ over tetrahedron $x = 0, y = 0, z = 0$ and $x + y + z \leq 1$.

Prove that ratio of volume of solid generated by revolving ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major and minor axis is $b : a$.

Find the volume of spherical cap of height h cut off from sphere of radius a .

Trace the following curve $r = a(1 + \cos \theta)$.

Find a point within a triangle such that sum of squares of its distance from three vertices is minimum.