

B.Sc. (Part-I) EXAMINATION – 2019
MATHEMATICS
Paper Second : Calculus

Note : Attempt questions from all Sections as per instructions.

Section – A (Very Short Answer Type Questions)

Note : Attempt all parts of this question. Give answer of each part in about 50 words. $1\frac{1}{2} \times 10 = 15$

1. (i) Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x, \forall a \in R.$
- (ii) Give an example of a continuous functions which is not differentiable.
- (iii) Evaluate $D^n a^x$, where $D = d/dx$.
- (iv) Write the statement of Euler's theorem on homogeneous functions.
- (v) Define Evolute and Involute.
- (vi) Evaluate $\int_0^{\pi/2} \sin^6 x dx.$
- (vii) Define Beta and Gamma functions.
- (viii) Write the formula for arc length in parametric form.
- (ix) Evaluate $\int_0^a \int_0^b (x^2 + y^2) dx dy.$
- (x) For the cardioid $r = a(1 - \cos \theta)$, Prove that $\phi = \frac{1}{2}\theta.$

Section – B (Short Answer Type Questions)

Note : Attempt all questions. Give answer of each question in about 200 words. $6 \times 5 = 30$

2. Evaluate the following limits :

(a) $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x}\right)^{yx^3}$ (b) $\lim_{x \rightarrow 0} \left(\frac{1}{1 - e^{yx}}\right).$

Or

If $y = x^2 e^x$, Prove that

$$y_n = \frac{1}{2}n(n-1)y_2 - n(n-2)y_1 + \frac{1}{2}(n-1)(n-2)y$$

3. Expand $2x^2 + 7x^2 + x - 1$ is power of $x - 2$.

Or

If $x^x y^y z^z = c$, show that at $x = y = z$,

$$\frac{\partial^2 z}{\partial x \partial y} = - \{x \log (ex)\}^{-1}.$$

4. Find the maximum value of u , where $u = \sin x \sin y \sin(x + y)$. Or
Find the asymptotes of the curve :

$$x^3 - 2y^3 + 2x^2y - xy^2 + xy - y^2 + 1 = 0.$$

5. If I_n denotes $\int_0^a (a^2 - x^2)^n dx$ and $n > 0$. prove that

$$I_n = \frac{2na^2}{2n+1} I_{n-1}.$$

Or

Find the area included between the curves :

$$y^2 = 4ax \text{ and } x^2 = 4by.$$

6. Find the volume of the solid generated by the revolution of the cissoid $y^2(2a - x) = x^3$ about its asymptote. Or

Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

Section - C (Long Answer Type Questions)

Note : Attempt any two questions. Give answer of each question in about 500 words.

10 × 2 = 20

7. If $u = x + y + z$, $v = xy + yz + zx$, $w = x^3 + y^3 + z^3 - 3xyz$. Show that u, v, w are not independent. Find a relation between them.

8. (a) Show that the pedal equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2b^2}.$$

- (b) Find the radius of curvature for the curve $r = 2a \cos \theta$.

9. Prove that the evolute of the tractrix $x = a(\cos t + \log \tan \frac{1}{2}t)$,

$$y = a \sin t, \text{ is the catenary } y = a \cosh \left(\frac{x}{a} \right).$$

10. Evaluate the integral $\iiint \sqrt{\frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}} dx dy dz$

Where integral is taken over all positive values of the x, y, z such that $x^2 + y^2 + z^2 \leq 1$.

11. Trace the curve $a^2y^2 = (2a - x)x^3$ and find the whole area of it.