BSC (PART – I) EXAMINATION, 2015

MATHEMATICS

Paper Third : Geometry & Vector Calculus

Note : Answer questions from all Sections as per instructions.

Section–A (Very Short Answer Type Questions)

Attempt all parts of this question. Give answer of each part in about 50 words. $1^{1/2} \times 10 = 15$

1. (i) Write down the condition under which $ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$ represents a circle.

(ii) Define conjugate diameters of a conic.

(iii) Write down the polar equation of a conic.

(iv) Define director circle of the conic.

(v) If a, b, c are direction ratio of a line, find its direction cosine.

(vi) If l_1, m_1, n_1 and l_2, m_2, n_2 are direction rations of two lines, then find out the direction ratios of the line perpendicular to these lines.

(vii) Write down the normal form of the equation of a plane.

(viii) Find the image of the point (1, 3, 4) in the plane

$$2x - y + z + 3 = 0.$$

(ix) Define central conicoids.

(x) Define gradient, Divergence and curl.

Section–B (Short Answer Type Questions)

Attempt all questions. Give answer of each question in about 200 words. $7 \times 5 = 35$

2. Prove that the point of intersection of two perpendicular tangents one to each of two given confocals lies on a circle. Or

If PSP' and QSQ' are perpendicular focal chord of a conic, prove that

$$\frac{1}{PS.SP'} + \frac{1}{QS.SQ'} = \text{constant}.$$

3. Prove that the straight lines whose direction cosines are given by the relation $al + bm + cn = 0$ and $\alpha l_2 + \beta m_2 + \gamma n_2 = 0$ are perpendicular if $a^2(\beta + \alpha) + b^2(\gamma + \alpha) + c^2(\beta + \gamma) = 0$ Or

Find the equation of the plane through $(-1, 1, 1)$ and $(1, -1, 1)$ perpendicular to the plane $x + 2y + 2z = 5$.

4. Find the equation of sphere which touches the plane $3x + 2y - z + 2 = 0$ at point $(1, -2, 1)$ and cuts the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$ orthogonally.

Or

Find the equation of cone whose vertex is (α, β, γ) and base $y^2 = 4ax, z = 0$.

5. Show that from a given point (α, β, γ), six normals in general can be drawn to an ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Or

Find the equation of a right circular cylinder of radius 2 whose axis is the line

$$\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{2}$$

6. State and prove Green's theorem in plane.

Or

Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at point $(1, 2, -1)$ in the direction $2\hat{i} - \hat{j} + \hat{k}$.

Section – C (Long Answer Type Questions)

Attempt any two questions. Give answer of each question in about 500 words. $10 \times 2 = 20$

7. Prove that the locus of the foot of the perpendicular from the focus of the conic $\frac{l}{r} = 1 + e\cos\theta$ on a tangent to it, is :

$$r^2(e^2 - 1) - 2ler\cos\theta + l^2 = 0$$

8. Find the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-2}{8} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

Find also its equations and its point of intersection with the given lines.

9. A sphere of constant radius K passes through the origin and meets the axis in A, B, C. Prove that the centroid of the angle ABC lines on the sphere.

$$9(x^2 + y^2 + z^2) = 4K^2$$

10. Prove that the locus of the foot of perpendicular from the centre of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ on the plane PQR is

$$a^2x^2 + b^2y^2 + c^2z^2 = 3(x^2 + y^2 + z^2)^2.$$

11. Evaluate integral $\int_C \vec{F} \cdot d\vec{r}$ where $F = (x^2 + y^2)\hat{i} - 2xy\hat{j}$

The curve C is the ractangle in they xy-plane bounded by $y = 0, x = a, y = b, x = 0$.

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