BSC (PART – I) EXAMINATION, 2017

MATHEMATICS

Paper Third : Geometry & Vector Calculus

Note : Answer questions from all Sections as per instructions.

Section – A (Very Short Answer Type Questions)

Attempt all parts of this question. Give answer of each part in about 50 words.

$1\frac{1}{2} \times 10 = 15$

(i) Write down the conditions under which the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ has its centre at origin.

(ii) Define confocal conics.

(iii) Find the direction cosines of the line which is equally inclined to the positive directions of the axes.

(iv) Define Plane.

(v) Define orthogonal spheres.

(vi) Define right circular cone.

(vii) Write down the standard equations of hyperboloid of one and two sheets.

(viii) Define the tangent plane and normal to a conicoid.

(ix) Write down the condition that the vector $a(t)$ has constant direction.

(x) State the theorem of Green in a plane.

Section – B (Short Answer Type Questions)

Attempt all questions. Give answer of each question in about 200 words.

$7 \times 5 = 35$

2. Find out the condition that m and m_1 are slopes of the conjugate diameters of the conic.

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Or

Prove that the equation of the confocal hyperbola through the point on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Whose eccentric angle is α, is $\dfrac{x^2}{\cos^2\alpha} - \dfrac{y^2}{\sin^2\alpha} = a^2 - b^2$

3. A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Prove that :

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$$

Or

Through a point $P(\alpha, \beta, \gamma)$ a plane is draw at right angle to OP to meet the axes in A, B, C, Prove the area of triangle $ABC = \dfrac{p^5}{2\alpha\beta\gamma}$ where $OP = p$.

4. Show that two spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y - 14z + 20 = 0$ are orthogonal.

Or

Prove that the equation of the cone, whose vertex is (0, 0, 0) and base is the curve

$$z = k, f(x, y) = 0 \text{ is } f\left(\frac{xk}{z}, \frac{yk}{z}\right) = 0$$

5. Find the equation of director sphere of the conicoid

$$ax^2 + by^2 + cz^2 = 1.$$

Or

Show that the pole of $lx + my + nz = p$ with respect to the conicoid $ax^2 + by^2 + cz^2 = 1$ is the point $\left(\frac{l}{ap}, \frac{m}{bp}, \frac{n}{cp}\right)$.

6. Prove that grad $(uv) = u$ grad $v + v$ grad u.

Or

Prove that :

$$\text{grad } (u, v) = u \times \text{curl } v + v \times \text{curl } u + (u.\nabla)\, v + (v, \nabla)\, u.$$

Section – C (Long Answer Type Questions)

Attempt any two questions. Give answer of each question in about 500 words. $10 \times 2 = 20$

7. Find the equation of the director circle of the conic

$$l/r = 1 + e \cos \theta.$$

8. Find the equation of the line which passes through the point (3, −1, 11) and is perpendicular to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ obtain the foot of the perpendicular.

9. Find the equation of the cylinder with generators parallel to z-axis and passing through the curve.

$$ax^2 + by^2 = 2z$$
$$lx + my + nz = p$$

Show that the six normals drawn from a given point (α, β, γ) to an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ lie on the cone

$$\frac{\alpha\,(b^2 - c^2)}{x - \alpha} + \frac{\beta\,(c^2 - a^2)}{y - \beta} + \frac{\gamma\,(a^2 - b^2)}{z - \gamma} = 1$$

State and prove Stokes theorem.