B. Sc. (Part-II) Examination 2014

MATHEMATICS First Paper
(Linear Algebra and Matrices)

Note:- Attempt questions from all sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Attempt *all* parts of this question. Give answer of each part in about 50 words.

$1^{1/2} \times 10 = 15$

1. (i) Let S be set of all ordered pairs of real numbers. Define sums and scalar multiples of pairs as follows:
$(x_1\, y_1)+(x_2\, y_2) = (x_1 + x_2\, ,\, y_1 + y_2)$
and
$c\,(x_1\, y_1) = (c\, x_1, y_1)$
Show that S is not a vector space.

(ii) Prove that:
$U = \{(a,b,c)$: a,b,c and ralionals$\}$
in not a subsace of $R^3(R)$.

(iii) Show that set $\{(1,2,1), (2,1,0), (1,-1,2)\}$ form a basis for $V_3\,(F)$

(iv) Define quotient space.

(v) Define range and null space of a linear transformation.

(vi) Define dual space and dual basis of a vector space.

(vii) Define rank of a matrix.

(viii) Write down the quadratic form corresponding to the symmetric marix.

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

(ix) Define inner product space.

(x) Define diagonalization of marix.

Section-B (Short Answer Type Questions)

Attempt *all* questions. Give answer of each question in about 200 words.

6x5=30

2. The union of two subspaces is a subspace if and only if one is contained in the other.

Or

Prove that the vectors (1,1,0), (3,1,3) and (5,3,3) are linearly dependent.

3. There exists a basis for each finite dimensional vactor space.

Or

Show that mapping $T:R^3 \rightarrow R^2$ defined by:

$$T\,(a,b,c) = (a,b)\ \forall\,(a,b,c) \in R^3$$

is a homomorphism of vector space $R^3\,(R)$ onto $R^2\,(R)$.

4. If $V(F)$ and $V'(F)$ are vector spaces and $T:V \rightarrow V'$ is a linear transformation, then the null space of T is a subspace of V.

Or

Find matrix of the linear tranformation $T:R^3 \rightarrow R^2$ given that:

$$T\,(x,y,z) = (x+y,\ x+z),\ \forall\,(x,\,y,\,z) \in R^3$$

relative to the basis {(1,0), (1,1)}.

5. The dual space of n dimensional vector space $V(F)$ is n dimensional i.e. dim V^*=dim$V$.

or

If x,y are any two vectors in an inner product space $V(F)$, then $|(x,y)| \leq \|x\| \|y\|$

6. Every square matrix can be uniquely expressed as the sum of a symmetric and skew symmetric matrix.

or

Find the rank of the matrix A where:

$$A=\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$$

Section-C (Long Answer type Questions)

Attempt any *two* questions. Give answer of each question in about 500 words.

10x 2=20

7. Find the characteristic roots and associated characteristic vectors for the matrix:

$$A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

8. Every finite dimensional inner product space has and orthonormal basis.

9. Verify that matrix:

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Satisfies Cayley-Hamilton theorem and hence find A^{-1}.

10. If W is a subspace of a finite dimensional vector space $V(F)$, then show that:

$$\dim W + \dim W^0 = \dim V.$$

11. Reduce the quadratic form to sum of squares:

$$q = 2x_2 x_3 - 2x_1 x_3 - 6x_1x_2 + x_1 x_4 - 4x_2 x_4 - 3x_3x_4$$

and find the transformation which actually affects the reduction.