B. Sc. (Part-II) Examination 2015

MATHEMATICS First Paper
(*Linear Algebra and Matrices*)

Note:- Attempt questions from all sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Attempt *all* parts of this question. Give answer of each part in about 50 words. $1^{1/2} \times 10 = 15$

1. (i) Define ring isomorphism.
   (ii) Define proper zero divisor with an example.
   (iii) If $V = R^3$, then show that $W = \{(x, y, 0); x, y, \in R\}$ is a subspace of V.
   (iv) Define Kernel of a linear transformation.
   (v) Define dual basis.
   (vi) If S is a non empty subset of a vector space V, then S° (the annihilator of S) is a subspace of V^*.
   (vii) Define inner product space.
   (viii) Define Skew-Hermitian matrix with example.
   (ix) Find the characteristic roots of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

   (x) Write down the quadratic form corresponding to the symmetric matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$$

Section-B (Short Answer Type Questions)

Attempt *all* questions. Give answer of each question in about 200 words. 6x5=30

2. Show that set W of the elements of the vector space $V_3(R)$, of the form $(x+2y, y, -x+3y), x, y \in R$ is a subspace of $V_3(R)$.

   Or

   In the vector space R^3 (R), express the vector (4,28,-4) as a linear combination of vectors (2,3,1), (-1,4,-2) and (1,18,-4) in two different ways.

3. If V is a finite dimensional vector, space, then any two basis have the same number of elements.

   Or

   Define linear sum of two subspace and prove that if W_1 and W_2 are two subspaces of a vector space $V(F)$, then the set $W_1+W_2 = \{x+y; x W_1, y \in W_2\}$ is also a subspace of $V(F)$.

4. Every n-dimensional vector space V over a field F is isomorphic to the vector space F^n.

   Or

Let T be the linear operator on R^2 defined by $T(x,y) = (4x - 2y, 2x + y)$. Compute the matrix of T relative to the basis (α_1, α_2) where $\alpha_1 = (1,1)$, $\alpha_2 = (-1,0)$.

5. Every bilinear form on a Vector supace V over a subfield F of complex number can be uniquely expressed as the sum of a symmetric and skew symmetric forms. (

Prove that in an inner product spapc V (F), the vectors x and y are linearly dependent iff:

$$|(x,y)| = \|x\| \cdot \|y\|$$

6. Reduce the following matrix in normal form and hence find the rank.:

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

Or

Find out for what value of , λ the following equations:

$$x + y + z = 1;\ x + 2y + 4z = \lambda;\ x + 4y + 10z = \lambda^2$$

have a solution and solve completely in each case.

Section-C (Long Answer type Questions)

Attempt any *two* questions. Give answer of each question in about 500 words.

10x 2=20

7. If W is a subspace of dimension m of a vector space $V(F)$ of dimersion n, then the dimension of quotient space V/W is n-m
i.e. dim (V/W) = dim V-dim W.

8. Verify that that matrix : $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

Satisfies Cayley-Hamilton theorem and hence find A^{-1}.

9. If $\{x_1, x_2, x_3.......x_m\}$ is any finite orthonormal set in an inner product space V and if y is any vector in V, then $\sum_{i=1}^{x} |(y,x_i)|^2 \leq \|y\|^2$
Further equality hold iff y belongs to the subspace spanned by $x_1, x_2, x_3.....x_m$.

10. Verify the following tranformation is a linear transformation:

$$T(a,b) = (a + b, a-b, b)\ \forall a,b \in R$$

Find the range space and null space.

11. Define basis of vector space. If B = { (-1, 1, 1), (1,-1,1), (1,1,-1) is a basis of $R^3(R)$ then find the dual basis of B.