B.Sc. (Part-II) Examination-2017

MATHEMATICS First Paper
(*Linear Algebra and Matrices*)

Note:- Attempt questions from all sections as per instructions.

Section-A
(Very Short Answer Type Questions)

Attempt *all* parts of this question. Give answer of each part in about 50 words. $1^{1/2} \times 10=15$

1. (i) Define Vector space.

(ii) Test the linear dependence independence of vector's (1, 2, 3), (2, -2, 0).

(iii) Define Isomorphism of vector space.

(iv) Define Annihilators.

(v) Define Rank and Nullity of Linear Transformation.

(vi) Define Dual Space.

(vii) If A be any matrix, then prove that AA' and A'A are both symmetric matrix.

(viii) Find the rank of matrix:

$$\begin{bmatrix} 5 & 10 \\ 3 & 6 \end{bmatrix}$$

(ix) State Cayley-Hamilton's theorem.

(x) Determine the eigen values of the matrix

$$A=\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & c & c \end{bmatrix}$$

Section-B (Short Answer Type Questions)

Attempt *all* questions. Give answer of each question in about 200 words. 6x5=30

2. Prove that union of two subspaces of a vector space V (F) is a subspace of V (F) iff one is contained in the other.

Or

In a finite dimensional vector space V (F) be the direct sum of its subspace U and W i.e. $V = U \oplus W$, then prove that

$$\dim (U \oplus W) = \dim U + \dim W.$$

3. If is a mapping of V_3 (F) onto V_2 (F) given by $f(x, y, z) = (y, z)$, then show that f is linear.

Or

If $T: V \rightarrow W$ where T is a linear Transformation.
Prove that range and null space of T are subspaces of W and V respectively.

4. If V_2 (R) is a vector and B = { (2,1), (3,1)} be a basis of V_2 (R), then find the dual basis of B.

Or

If S is any subset of vector space V (F) then Show that S^0, the annihilator of S is a subspace of V (F).

5. Find the rank of the matrix

$$A=\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

Or

Show that the equations

$2x - y + 3z = 8$

$x + 2y + z = 4$

$3x + y - 4z = 0$

are consistent and solve them.

Show that every square matrix is uniquely expressible as the sum of a Hermition matrix and skew Hermitian matrix. Or

Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}$$

section-C (Long Answer type Questions)

Attempt any two questions. Give answer of each question in about 500 words. 10x 2=20

If W be a subspace of a finite dimensional vector space V (F) than dim V/W = dim V - dim W Where V/W is the quotient space

$$V/W = \{ W+\alpha : \alpha \in V \}.$$

If T be a linear transformation of n- dimensional vector space V (F), then prove that

Rank of T + Nullity of T = n.

If W_1 and W_2 are subspaces of a finite dimensional vector space V (F) then show that

(i) $(W_1 \cap W_2)^0 = W_1^0 + W_2^0$ (ii) $(W_1 + W_2)^0 = W_1^0 \cap W_1^0$

Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

by using E- transformation.

Find the characteristic equation of martix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and Verify that it is satisfied by A and hence obtain A^{-1}.