B.Sc. (Part-II) Examination-2019

MATHEMATICS First Paper
(*Linear Algebra and Matrices*)

Note:- Attempt questions from all sections as per instructions.

Section-A
(Very Short Answer Type Questions)

Attempt *all* parts of this question. Give answer of each part in about 50 words. $1^{1/2} \times 10 = 15$

1. (i) Show that set { 1, 2, 3}, (2, 1, 0), (1, -1 , 2)} forms a basis for V_3 (F.)

(ii) Define quotient space.

(iii) Define null space of a linear transformation.

(iv) What do you mean by rank and nullity of linera transformation?

(v) Define Inner product space.

(vi) Define bilinear form. Give example.

(vii) Prove that

$$\beta = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix} \text{is unitary.}$$

(viii) If A and B are symmetric matrices, show that AB is symmetric.

(ix) Show that the characteristic roots of skew Hermitian matrix are either pure imaginary or zero.

(x) Find the characteristics equation of

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}.$$

Section-B (Short Answer Type Questions)

Attempt *all* questions. Give answer of each question in about 200 words. $6 \times 5 = 30$

2. The Union of two subspaces is a subspace iff one is contained in the other.

or

Prove that

$\dim \frac{V}{W} = \dim V - \dim W$ where W is a subspece of a finite dimensional vector space V (F).

3. Let U and V be a vector space over the field F and let T be a linear transformation form U into V. IF U is a finite dimensional, then prove that rank (T) + nullity (T) = dim U.

Or

Find matrix of the linear transformation $T: R^3 \to R^2$ given that: $T(x, y, z) = (x + y, x + z), \forall (x, y, z) \in R^3$ relative to the basis {(1,0), (1,1)}.

4. State and prove Cauchy-Schwarz's in equality.

Or

Find the dual basis of the basis set $B = \{ (1, -2, 3), (1, -1, 1), (2, -4, 7) \}$ of V_3 (R).

Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

Or

Find the eigen values and corresponding eigen vectors of the matrix

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

6. Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrices.

Or

Determine the characteristic roots and corre-sponding characteristic vectors of the following matrix:

$$A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Section-C (Long Answer type Questions)

Attempt any *two* questions. Give answer of each question in about 500 words.

10x 2=20

7. If W_1 and W_2 are finite dimensional space of a vector space V (F), then prove that

$$\dim (W_1 + W_2) = \dim W_1 + \dim W_2 - \dim (W_1 \cap W_2).$$

8. Prove that every finite dimensional inner product space has an orthogonal basis.

9. Show that the matix:

$$A=\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

is diagonalizable. Also find the diagonal matrix and the transformation matrix.

10. Investigate for what value of λ and μ the equations:

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

have:

(i) no Solution (ii) unique solution

(iii) an infinity of solution.

11. Verify that matrix:

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Satisfies Caley-Hamilton theorem and Hence find A^{-1}

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