BSC (PART – II) EXAMINATION, 2015

MATHEMATICS

Third Paper-2015
(*Mechanics*)

Note:- Attempt questions from all sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Attempt *all* parts of this question. Give answer of each part in about 50 words. $1^{1/2} \times 10 = 15$

1. (i) Write the formula of radial and transverse accelerations of a particle moving in a plane.
   (ii) Define S.H.M. and write the formula for the time taken in one complete oscillation.
   (iii) Define central axis of forces acting on a body.
   (iv) Find the work done by the tension T of an extensible string of length l during a small displacement.
   (v) Define angular velocity and angular acceleration of a particle moving in a plane.
   (vi) Define a constrained motion'
   (vii) Define reciprocal screws.
   (viii) Define span, sag and directrix of a catenary.
   (ix) Define null point, null plane and null line.
   (x) Define stable, unstable and neutral equilibrium of a body.

Section-B (Short Answer Type Questions)

Attempt *all* questions. Give answer of each question in about 200 words. $7 \times 5 = 35$

2. A particle moves alsong a circle $r = 2a\cos\theta$ in such a way that its accelaration towards the origin is always zero. Show that the transverse acceleration varies as the fifth power of cosec θ.

   Or

   A particle describes a cycloid with uniform speed. Prove that the normal acceleration at any point varies inversely as the square root of the distance from the base of the cycloid.

3. A particle moves along the axis of x starting from rest at $x = a$. For an interval t_1 from the beginning of the motion the acceleration is $-\mu x$, for a subsequent t_2 the acceleration is μx and at the end of this interval the particle is at the origin; prove that:

$$\tan(\sqrt{\mu}\, t_1) \tan h(\sqrt{\mu}\, t_2) = 1$$

Or

Two light elastic strings are fastened to a particle of mass m and their other ends to fixed points so that the strigs are taut. The modulus of each is λ, the tenstion T and lengths a and b. Show that the period of an oscillation along the line of the strings is

$$2\pi\left[\frac{mab}{(T+\lambda)(a+b)}\right]^{1/2}$$

4. A particle is placed on the outside of a smooth vertical circle. If the particle starts from a point whose angular distance is α from the highest point of the circle show that it will fly off the curve when $\cos\theta = \frac{2}{3}\cos\alpha$ Or

A particle projected upwards with a velocity U, in a medium whose resistance varies as the square of the velocity, will return to point of projection with velocity $V_1 = \frac{UV}{\sqrt{U^2+V^2}}$ after a time $\frac{V}{g}\left(\tan^{-1}\frac{U}{V} + \tanh^{-1}\frac{v_1}{V}\right)$ Where V is the terminal velocity.

5. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log\left\{\frac{1+\sqrt{1+\mu^2}}{\mu}\right\}$ where μ is the coefficient of friction. Or

Equal forces act along the coordinate axes and the straight line.

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

Find the equation of the central axis of the system.

6. A hemisphere rests in equilibrium on a sphere of equal radius; show that the equilibrium is unstable when the curved, and stable when the flat surface of the hemisphere rests on the sphere. Or

Two uniform rods AB and Ac smoothly jointed at A are in equilibrium in a vertical plane. B and C rest on a smooth horizontal plane and the middle points of AB and AC are connected by a string. Show that the tension of the string is $\frac{W}{\tan B + \tan C}$ where W is the weight of the rods AB and AC.

Section-C (Long Answer type Questions)

Attempt any *two* questions. Give answer of each question in about 500 words $10 \times 2 = 20$

7. A particle is projected vertically upwards under gravity, supposed constant, in

resisting medium whose resistance varies as the velocity; discuss the motion.

8. Show that the length of an endless chain which will hand over a circular pulley of radius a so as to be in contact with two-thirds of the circumference of the pulley is $a\left\{\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right\}$

9. A particle is placed very near the vertex of a smooth cycloid whose axis is vertical and vertex upwards and is allowed to rundown the curve. Prove that it will leave the curve when it has fallen through half the vertical height of the cycloid. Also prove that the latus rectum of the parabola subsequently described is equal to the height of the cycloid.

10. A particle describes the curve $= r^n = A\cos n\theta + B\sin n\theta$ under a force to the pole. Find the law of the force.

11. A force P acts along the axis of X and another force nP along a generator of the cylinder $x^2 + y^2 = a^2$. Show that the central axis lies on the cylinder $n^2(nx-z)^2 + (1+n^2)^2 y^2 = n^4a^2$