BSC (PART – II) EXAMINATION, 2017

MATHEMATICS

Third Paper-2017
(*Mechanics*)

Note:- Attempt questions from all sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Attempt *all* parts of this question. Give answer of each part in about 50 words. $1\frac{1}{2} \times 10 = 15$

(i) Define the terms Radial and Normal accelerations.

(ii) Explain the Hooke's Law for elastic materials.

(iii) A particle is describing an ellipse under a force to a pole, find differential equation for its motion.

(iv) Define the term terminal velocity for resisting medium. It a particle is projected upwards with a velocity 'u' in a medium whose resistnce varies as the velocity.
Write the differential eq. for its acceleration.

(v) Write down conditions of Equilibrium of a body under the action of coplaner forces.

(vi) Write the relation between ρ and φ for a common catenary. Explain about sysbols used there in.

(vii) Define Wrench and intensity of Wrench.

(viii) Write the equation to the null plane of a given point (f,g,h) referred to Ox, OY, & OZ axes.

(ix) Define the terms Virtual displacement and Virtual Work.

(x) Differentiate between Kinematics and Kinetics.

Section-B (Short Answer Type Questions)

Attempt *all* questions. Give answer of each question in about 200 words. $7 \times 5 = 35$

A point executes S.H.M. Such that in two of its positions, the velocities are u,v and the corresponding accelerations are α, β ;show that the distance between the positions is

$$\frac{V^2 - u^2}{\alpha + \beta}$$

and the amplitude of the motion is

$$\frac{\sqrt{(v^2 - u^2)(\alpha^2 v^2 - \beta^2 u^2)}}{(\beta^2 - \alpha^2)}$$

Or

A particle is describing a plance curve. If the tangential and normal accelerations are each constant through out the motion, prove that the angle ψ, through which the direction of motion truns in time t, is given by

$$\psi = A \log (1 + B.t).$$

3. An elastic string without height of which unstretched length is ℓ and modulus of elasticity is 'n'. The string is supended by one end and a mass 'm' is attached to other end. Show that the time of a small vertical oscillation is

$$2\pi\sqrt{\frac{m}{ng}}$$

Or

If h be the height due to velocity v at the earth's surface supposing its attraction constt., and H the corresponding height when the variation of gravity is taken into account, prove that

$$\frac{1}{h} - \frac{1}{H} = \frac{1}{r}$$

where r is the Earth's radius.

4. A particle describes a parabola $2p^2 = \ell r$ under a force to its pole. Derive the Law of force.

Or

If the ends of a uniform in extensible string of length ℓ hanging freely under gravity slides on a fixed rough horizontal rod whose coefficient of friction is μ, show that at most they can rest at a distance

$$\mu\ell \log\left[\frac{1 + \sqrt{1 + \mu^2}}{\mu}\right]$$

5. Find the centre of gravity of a cone of height h and base radius r.

Or

A heavy uniform cube balances on the highest point of a sphere whose radius is a. If the sphere is rough enough to prevent sliding and if the side of the cube be $\pi a/2$, show that the cube can rock through a right angle without falling.

6. A particle moves in a smooth sphere under no forces except the pressure of the surface. Show that its path is given by the equation

$$\text{Cot } \theta = \cot \beta \cdot \cos \phi,$$

where θ, ϕ are its angular Coordinates.

Or

A particle of mass m is falling under the influence of gravity through a medium whose resistance is equal to μ times the velocity. If the particle were released from rest, Show that the distance fallen through in time t is,

$$\frac{m^2 g}{\mu^2}\left[\frac{\mu . f}{m} + e^{-\left(\frac{\mu}{m}\right)t} - 1\right]$$

Section-C (Long Answer type Questions)

Attempt any *two* questions. Give answer of each question in about 500 words

10x 2=20

7. Three forces act along the straight lines

$$x = 0,\ y - z = a;$$
$$y = 0,\ z - x = a;$$
$$z = 0,\ x - y = a;$$

Show that they can not reduce to a couple. Prove also that if the system reduce to a single force, its line of action must be on the surface

$$x^2 + y^2 + z^2 - 2yz - 2zx - 2xy = a^2$$

8. A heavy elastic string whose natural length is $2\pi a$, placed round a smooth cone whose axis is vertical and whose semi-vertical angle is α. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium in the form of a circle whose radius is,

$$a\left(1+\frac{W}{2\pi\lambda}\operatorname{Cot}\alpha\right)$$

9. A particle, subject to a central force per unit of mass equal to

$$\mu\{2(a^2+b^2)u^5 - 3a^2b^2u^2\}.$$

is projected at the distance with a velocity $1/a\sqrt{\mu}$ in a direction at right angles to the initial distance, show that the path is the curve

$$r^2 = a^2\cos^2\theta + b^2\sin^2\theta$$

10. A particle is projected vertically upwards under gravity with velocity V the resistance of the air produces a retardation Kv^2, where v is the velocity. Show that the velocity V' with which the particle will return to the point of projection is given by

$$\frac{1}{(v')^2} = \frac{1}{V^2} + \frac{K}{g}$$

11. A particle is projected along the inner surface of a rough sphere and is acted on by no forces. Show that it will return to the point of projection at the end of time

$$\frac{a}{\mu v}(e^{2\mu\pi}-1),$$

Where a is the radius of the sphere, V is the velocity of projection and μ is the coeff. of friction.