BSC (PART – II) EXAMINATION, 2019

MATHEMATICS

Third Paper-2019

(*Mechanics*)

Note:- Attempt questions from all sections as per instructions.

Section-A (Very Short Answer Type Questions)

Attempt *all* parts of this question. Give answer of each part in about 50 words. $1^{1/2} \times 10 = 15$

1. (a) Define radical and transversal velocities.
   (b) A particle describe a parabola with uniform speed. Show that its angular velocity about the focus at any point P varies inversely as $(Sp)^{3/2}$.
   (c) Define amplitude and frequency in SHM.
   (d) State Hooke's Law for elastic strings.
   (e) If $(\bar{x}, \bar{y})$ be the co-ordinate of the C.G of the area of the first quadrant of the ellipse , $\frac{x^2}{a^2}+\frac{y^2}{b^2}= 11$, the find $(\bar{x}, \bar{y})$
   (f) Define Common Catenary and vertex of the catenary.
   (g) Define Virtual displacement and virtual work.
   (h) Define stable and Neutral equilibrium.
   (i) Define poinsot's Central axis.
   (j) Define Null lines, Null Plane and Null point.

Section-B (Short Answer Type Questions)

Attempt *all* questions. Give answer of each question in about 200 words. $7 \times 5 = 35$

2. A particle moves is a curve $y = a \log \sec\left(\frac{x}{a}\right)$ in Such a way that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of the radius of curvature. Or
   A point moves in the plance curue so that its tangential and normal acceleration are equal to angular velocity of the tangent is constant. Find the curve.
3. A particle is moving with SHM of amplitude 'a' and periodic time T.
   Prove that $\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}$ Or
   A particle describes the curve $r^n = a^n \cos n\theta$ under a force to the pole. Find the Law of force.

4. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log\left[\frac{1+\sqrt{1+\mu^2}}{\mu}\right]$ where μ is coefficient of friction. Or

If α, β be the inclination to the horizon of the tangents at the extremities of a portion of a common catenary and l the length of the portion show that the height of one extremity above the other is $\frac{l \sin\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\alpha-\beta)}$ the two extremities being on one side of the vertex of the catenary.

5. The necessary and sufficient condition that a system of coplanar forces acting on a particle or a rigid body in equilibrium is that the algebraic sum of the virtual works done by the forces during a small displacement consistent with the geometrical conditions of the system is zero to the first degree of approximation. Or

Two uniform rods AB and Ac Smoothly Joined, at A are in equilibrium in a vertical plance, B and C rest on a smooth horizontal plane and the middle points of AB and AC are connected by a string. Show that the tension of string is $\frac{W}{\tan B + \tan C}$ where W is the total weight of rods AB and AC.

6. Find the null point of the plane $x + y + z = 0$ for the force system (X,Y,Z; L, M, N). Or

A system of forces given by (X, Y, Z; L, M, N) is replaced by two forces one acting along the axis of x and other free. Show that the magnitudes of the forces are $\frac{LX + MY + NZ}{L}$ and $\frac{\sqrt{(My+NZ)^2 + L^2(Y^2+Z^2)}}{L}$

Section-C (Long Answer type Questions)

Attempt any two questions. Give answer of each question in about 500 words $10 \times 2 = 20$

7. A particle falls from rest in a medium in which the resistance is kv^2 per unit mass. Prove that the distance fallen in time t is

$$\frac{1}{k} \log \cos h \{ t\sqrt{gh} \}.$$

8. A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest, prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half.
9. If w be the angular velocity of a planet at the nearer end of major axis, prove that its period is $\frac{2\pi}{w}\sqrt{\frac{1+e}{(1-e)^3}}$
10. Find the position of the centre of gravity of the arc of the cardiod $r = a(1 + \cos\theta)$ lying above the initial line.
11. A force P acts along the axis of x and another force nP along a generator of the cylinder $x^2 + y^2 = a^2$. Show that the central axis lies on the cylinder $n^2(nx - z)^2 + (1 + n)^2 y^2 = n^4 a^2$.