B.Sc. (Part-III) Mathematis Examination–2020

Mathematics (Fourth (C) Paper)

(Differential Geometry and Tensor Analysis)

Note : Attempt questions from all sections as per instructions

[Section–(A) Very Short Answer Type Questions]

Note : Attempt all parts of this question. Give answer of each question in about 50 words **$1\frac{1}{2} \times 10 = 15$**

1. (a) Define circular helix
   (b) Define first fundamental form
   (c) Define surface of revolution
   (d) Define normal curvature
   (e) Define Mean and Gaussian Curvature.
   (f) State Mainardi-Codazzi equation.
   (g) How many components does a tensor of rank 3 have in a space of 4 dimension ?
   (h) Define Riemannian curvature tensor
   (i) State fundamental theorem of Riemannian geometry.
   (j) State quotient law of tensor.

[Section–(B) Short Answer Type Questions]

Note : Attempt all question. Give answer of each question in about 200 words **$8 \times 5 = 40$**

2. Prove that the osculating plane at a point P has, in general, three point contact (Contact of second order) with the curve at P.

   Or Show that the involutes of a circular helix are plane curves.

3. The metric or first fundamental form is invarient under a transformation of parameters.

   Or State and prove Meusniers Theorem.

4. Show that the tensor product of two tensors of type (r, s) and (r', s') is tensor of the type $(r+r', s+s')$.

   Or If K, K_n and K_g denote curvature, normal curvature and geodesic curvature of a curve on the surface, prove that $K^2 = K_n^2 + K_g^2$.

5. A necessary and sufficient condition that the curve of vector field vanishes is that the vector field be gradient.

   Or Show that :

   $$\text{div } A^i = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} (A^i \sqrt{g}) \text{ where } g = |g_{ij}|$$

6. Determine the number of independent components of Christoffel symbols.

   Or If connection is symmetric and compatible with metric g_{ij} then prove that $R_{ijkl} = R_{klij}$.

[Section–(C) Long Answer Type Questions]

Note : Attempt any two questions. Give answer of each question in about 500 words. **10 × 2 = 20**

7. State and Prove the Gauss-Bonnet Theorem.

8. Find the evolute of the $\bar{r} = \bar{r}(s)$.

9. Find the Mean and Gaussian Curvature at a point of the surface $\bar{r} = (u\cos v, u\sin v, cv)$.

10. Prove that the covarient derivative of tensors g_{ij}, g^{ij} and δ_{ij} all vanish identically.

11. State and Prove Euler's Theorem.