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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

First Paper

(Abstract Algebra)

Time : Three Hours] *[Maximum Marks : 80*

Note : Attempt questions from **all** Sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. Give answer of each part in about 50 words. $1\times10=10$

1. (i) Let G be a group and $a\in G$ be fixed element. Prove that a mapping $f:G\to G$ given by $f(x) = a^{-1}xa \forall x \in G$ is an automorphism.

P.T.O.

(ii) Define centre of group and conjugate subgroup.

(iii) If S is an ideal of R and $1\in S$, prove that $S=R$.

(iv) Define Fuclidean ring and give one example.

(v) Define simple group and show that a group of order 105 is not simple.

(vi) Define finite extension of field. Show that there can be a finite field containing 36 elements.

(vii) State the fundamental theorem of Galois theory.

(viii) Define an Algebraic element in an extension filed.

(ix) Define uniform and primary modules.

(x) Define irreducible module and simple module.

Section-B

Note : Attempt **all** questions. Give answer of each question in about 200 words. Each question carries 10 marks. $10\times5=50$

2. If a group G has only one p-sylow subgroup H, then prove that, H is normal subgroup of G.

OR

If z denote the centre of a group G. Then show that $a \in z$ iff $N(a)=G$. Also show that if G is finite, then $a \in z$ iff $O(N(a))=O(G)$.

3. If R is unique factorization domain, then prove that R[x] is also UFD.

OR

Prove that a subgroup of a solvable group is solvable.

4. Show that the ring of polynomials over a field of real is an Euclidean ring.

OR

Find all composition series of Z_{60} (the group of integers under addition modulo 60).

5. If L is an algebraic extension of and K is algebraic extension of field F, then prove that L is an algebraic extension of F.

OR

Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

6. Prove that an Irreducible R-modules in cyclic.

OR

Prove that every submodule of an Artinian module Is Artinian.

Section-C

(Long Answer Type Questions)

Note : Attempt any **two** questions. Give answer of each question in about 500 words. 10×2=20

7. Let G be a group of finite order and p be a prime number if $p^m/o(G)$ and p^{m+1} is not a divisor of o(G), then G has a sub group of order p^m.

8. Show that

$$\frac{G}{z(G)} \cong I(G)$$

Where (G,O) is a group, z(G) is the centre of the group and I(G) is the subgroup of all inner automorphisms of G.

9. State and prove Jordan Holder theorem.

10. Prove that the Galois group of a polynomial orver a field F is isomorphic to a group of permutations of the roots of the polynomial.

11. State and prove Noether and Lasker theorem.

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