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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Sixth (A, B, C, D & E) Paper

Sixth (A) Paper

(Differential Equations)

Time : Three Hours] *[Maximum Marks : 100*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. Give answer of each part in about 50 words. Each part carries 2 marks. 2×10=20

P.T.O.

1. (i) Solve the initial value problem.

$y'=y^2$, $y(1) = 2$

(ii) Find the general solution of

$$x^2y'' + 7xy' + 13y=0$$

(iii) State Kneser's theorem.

(iv) State Kamke's Convergence theorem.

(v) Define Linear system with periodic coefficients.

(vi) Show that $y'' + 4y=0$, $y(0) = 0$ and $y(\pi) = 0$ has many non-trivial solutions.

(vii) Define Nodes and Saddle points.

(viii) Define Aprorri bounds.

(ix) Discuss Lyamunov functions.

(x) Define Index of stationary point.

Section-B

(Short Answer Type Questions)

Note : Attempt **all** questions. Give answer of each question in about 200 words. Each question carries two marks. $10 \times 5 = 50$

2. State and prove Peano's existence theorem.

OR

Examine the existence and uniqueness of the solution of initial value problem

$$\frac{dy}{dx} = y^2 \quad , \quad y(1) = -1,$$

3. Find the Eigen values of the boundary value problem

$$y'' + \lambda y = 0, \; y(0) = 0, \; y(1) = 0.$$

OR

If $f \in C$ on the rectangle R, then prove that there exists a solution $\phi \in C$ of (E) on

$$|t - T| \leq \alpha \text{ for } \phi(T) = \xi$$

4. Transform the equation

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

into an equivalent self adjoint equation.

OR

State and prove Ascoli-Arzela theorem.

5. Explain the Naguma's and Osgood's Criteria with illustration.

OR

State and prove the theorem of Winter.

6. Form an integral equation from the differential equation

$$y'' - 4y = 12$$

with boundary conditions

$$y(0) = 4, \; y'(0) = 1.$$

OR

If the critical point (0,0) of the linear plane autonomous system :

$$\frac{dx}{dt} = ax + by \ ,$$
$$\frac{dy}{dt} = cx + dy$$

is strictly stable, the s_0 is that of the perturbed system.

$x = ax+by+\xi\ (x,y),\ y = cx+dy+\eta(x,y);$

provided that

$$|\xi\ (x,y)| + |\eta(x,y)| = 0\ (x^2+y^2)$$

Section-C

(Long Answer Type Questions)

Note : Attempt any **two** questions. Give answer of each question in about 500 words. Each question carries 15 marks.

$15\times2=30$

7. State and prove Floquet' theorem.

8. Write in detail on maximal intervals of existence giving the extension theorem with its proof.

9. Determine proper nodes of the system

$$\frac{dx}{dt} = -x \qquad \frac{dy}{dt} = -ky$$

where k is constant.

10. Prove that any solution of

$x'' + a(t)x' + b(t)\ x= 0,\ t \geq 0$

has almost complete number of zeros in $(0, \infty)$.

11. State and prove Sturm Oscillation Theorem.