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M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Fifth Paper

(Integral Equation and Boundary Value Problem)

Time : Three Hours] *[Maximum Marks : 80*

Note : Attempt questions from **all** sections as per instructions.

Section-A

(Very Short Answer Type Questions)

Note : Attempt **all** parts of this question. Each part carries **2** marks. 2×10=20

1. (i) Define linear and non-linear integral equation

   (ii) Write a Fredholm integral equation of second kind.

(iii) Define homogenous Volterra integral of the second kind.

(iv) Define symmetric Kernal with example.

(v) Write Leibnitz's rule of differentiation under integral sign.

(vi) Show that $y(x) = 1-x$ is a solution of the integral equation

$$\int_0^x e^{x-t} y(t)\,dt = x$$

(vii) Solve the Volterra integral equation of the first kind;

$$\int_0^t y(x)\,y(t-x)\,dx = 16\sin ut$$

(viii) Define Green function.

(ix) What do you mean by Neumann series.

(x) Define Dirac delta function.

Section-B

(Short Answer Type Questions)

Note : Attempt **all** questions. Each question carries **8** marks. 8×5=40

2. Show that the function $y(x)= (1+x^2)^{-3/2}$ is a solution of the Volterra integral equation

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)dt$$

OR

Convert the following differential equation into integral equation $y''+y=0$ with initial condition $y(0)= y'(0)= 0$

3. Using iterative method, solve

$$y(x) = f(x) + \lambda \int_0^1 e^{x-t} y(t)dt$$

OR

Find the eigen value and corresponding eigen function of the homogeneous integral equation

$$y(x) = \lambda \int_0^1 \sin \pi x \cos \pi t \, y(t)dt$$

4. Solve the Volterra integral equation of second kind by Laplace transform

$$y(x) = x^2 + \int_0^x y(u) \sin(x-u)du$$

OR

Solve the integral equation

$$\int_0^\infty f(x) \cos px dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$$

5. Solve the Fredholm integral equation

$$y(x) = \cos x + \lambda \int_0^\pi \sin x \, y(t)dt$$

OR

Solve the integral equation

$$x = \int_0^x \frac{y(t)\,dt}{(x-t)^{1/2}}$$

6. Find the iterated Kernel of the function whose kernel is given by

$K(x, t) = e^x \cos t$, when $a=0$, $b=\pi$

OR

Using method of succesive approximation, solve the integral equation:

$y(x) = 1 + \int_0^x y(t)\,dt$ taking $y_0(x)=0$

Section-C

(Long Answer Type Questions)

Note : Attempt any **two** questions. Each question carries **10** marks. $10\times2=20$

7. Obtain Fredholm integral equation to the boundary value problem $\frac{d^2u}{dx^2} + \lambda u = x$ with boundary condition $u(0)= u'(0)= 0$. Also, recover the B.V.P from the integral equation you obtain.

8. Solve $y(x) = f(x) + \lambda \int_0^1 (1-3xt)\,y(t)\,dt$

9. Find the Green is function for the boundary value problem $\frac{d^2y}{dx^2} + w^2y = 0$, $y(0)=y(1)=0$

10. Show that the integral equation

$$y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+t)\,y(t)\,dt$$

possesses no solution for $f(x)=x$ but it possesses infinitely many solutions with $f(x)=1$.

11. (a) Solve the Abel's equation

$$\int_0^t \frac{y(x)\,dx}{(t-x)^{1/3}} = t(1+t)$$

by Laplace transform.

(b) Solve:

$$\int_0^\infty F(x)\cos px\,dx = e^{-p}$$